Youngstown State University
Department of Mathematics
Fall 2003

Problem Solving Seminar 6

1. Can you find positive integers m and n, such that

$$m^2 + (m+1)^2 = n^4 + (n+1)^4 \ ?$$

2. Let f be a real function, such that for every real x,

$$f(x+1) + f(x-1) = \sqrt{2}\, f(x).$$

   Show that the function is periodic.

3. Define the infinite sequence as follows: $a_1 = 5$ and

$$a_{n+1} = \begin{cases} \dfrac{a_n}{2}, & \text{if } a_n \text{ is even} \\[2ex] \dfrac{a_n + 2003}{2}, & \text{if } a_n \text{ is odd} \end{cases}$$

   Show that $a_n = 5$ for infinitely many values of n.

4. Decide whether $\sqrt[3]{\sqrt{5}+2} - \sqrt[3]{\sqrt{5}-2}$ is a rational number.

5. Let S be a set of points in space. We can extend the set by adding points obtained by reflecting a point from S through another point belonging to S. Suppose that initially the set S consists of seven vertices of a cube. Can one ever get the eighth vertex of the cube into the set S by repeating the above operation?