

Youngstown State University  
Department of Mathematics  
Fall 2003

Problem Solving Seminar 5

1. Let  $\{x\}$  be the integer nearest to  $x$ . Show that  $m + \{\sqrt{m}\}$  can never be a square if  $m$  is a positive integer.
2. If the sum of the face angles at each vertex of a tetrahedron is  $180^\circ$ , prove that the opposite edges are equal in pairs.
3. Let  $a, b$  and  $c$  be pairwise distinct integers and let

$$f(x) = \frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-a)(x-c)}{(b-a)(b-c)} + \frac{(x-b)(x-c)}{(a-b)(a-c)}$$

Show that  $|f(a+b+c)| \leq a^2 + b^2 + c^2$ .

4. A convex quadrilateral is cut by its diagonals into four triangles with integer areas. Show that the product of the four areas of the triangles is a square of an integer.
5. Let  $n \geq 2$  and  $A = \{1, 2, \dots, n\}$ . Show that there exists a subset  $B$  of  $A$  having at most  $2\lfloor\sqrt{n}\rfloor + 1$  elements, such that

$$\{|x-y| \mid x, y \in B, x \neq y\} = \{1, 2, \dots, n-1\}.$$