Youngstown State University
Department of Mathematics
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Problem Solving Seminar 4

1. Show that 1280000401 is a composite integer.

2. Each edge of a 2003 vertex polyhedron is assigned a number $+1$ or -1. Does there always exist a vertex such that the product of the numbers on all edges meeting at that vertex is equal $+1$?

3. Each of m cards is labelled by one of the numbers $1, 2, \ldots, m$, where each label can be used more than once. Prove that if the sum of the labels of any subset is not a multiple of $m + 1$, then each card is labelled by the same number.

4. If corresponding skew (non-intersecting) edges of a tetrahedron are of equal length, prove that the centers of the inscribed and circumscribed spheres coincide.

5. For integers $n \geq 1$ define the sequence $a_n = \lfloor n\sqrt{2} \rfloor$. Show that the sequence contains infinitely many powers of 2.