Youngstown State University
Department of Mathematics
Fall 2003

Problem Solving Seminar 3

1. Show that for any integer $n \geq 1$ the number of binomial coefficients $\begin{pmatrix} n \\ k \end{pmatrix}$ which are odd integers is of the form 2^r.

2. For $n = 1, 2, ...$ let $s(n)$ denote the sum of the digits of 2^n. Thus for example as $2^8 = 256$, then $s(8) = 2 + 5 + 6 = 13$. Determine the integers n such that $s(n) = s(n+1)$.

3. For $x > 1$ evaluate

$$S(x) = \frac{x}{(x+1)} + \frac{x^2}{(x+1)(x^2+1)} + \frac{x^4}{(x+1)(x^2+1)(x^4+1)} + ...$$

4. Find two 2×2 matrices B and C with integer entries such that:

$$\begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} = B^3 + C^3$$

5. Find all pairwise relatively prime positive integers a, b and c such that

$$(a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

is an integer.