

Youngstown State University
Department of Mathematics
Fall 2003

Problem Solving Seminar 1

1. Show that no integer of the form $xyxy$ in base 10 is the cube of an integer.
2. Let $x > 1$, $y > 1$, $z > 1$, and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$. Show that

$$\sqrt{x+y+z} \geq \sqrt{x-1} + \sqrt{y-1} + \sqrt{z-1}.$$

3. Two matching decks have 36 cards each. One deck is shuffled and put on top of the other. For each card in the top deck we count the number of cards between it and the corresponding card in the bottom deck. What is the sum of these numbers?
4. Let $p(x)$ be a monic polynomial with integer coefficients such that for some distinct integers a, b, c , and d we have $f(a) = f(b) = f(c) = f(d)$. Does there exist an integer k with $f(k) = 25$? (Note: a *monic* polynomial has leading coefficient 1.)
5. A mathematics teacher wrote the quadratic $x^2 + 10x + 20$ on the blackboard. Then each student either increased or decreased by 1 either the linear coefficient or the constant term. Finally the quadratic $x^2 + 20x + 10$ appeared. Did a quadratic with integer coefficients necessarily appear in the process?